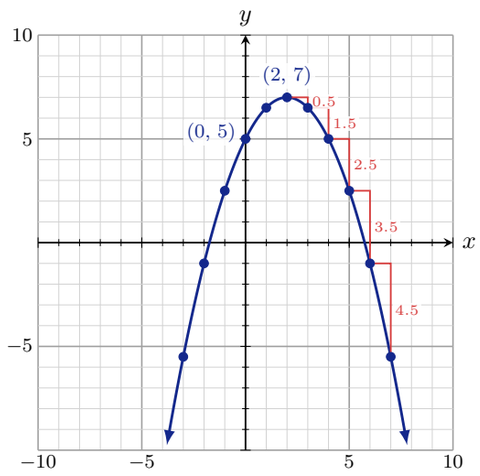
Practice Test – Quadratic Relations

/K 6 /T 6 /C 6 /A 6

Practice · Full Solutions

Marks *For all questions, use good form. Show your work: do not simply write a final answer.*

K 3 1. Sketch $y = -\frac{1}{2}(x-2)^2+7$ on the grid. Label the vertex and the y-intercept. Then complete each line.



Vertex: (2, 7)

Axis of symmetry: $x = 2$

y-intercept:

$y = -\frac{1}{2}(0-2)^2+7$ substitute $x = 0$
$= -\frac{1}{2}(4)+7$
$= -2+7$
$= 5$

The y-intercept is 5.

Review: *Transformations of the Quadratic Relation* and *Analysing Vertex Form and Equations from Graphs*

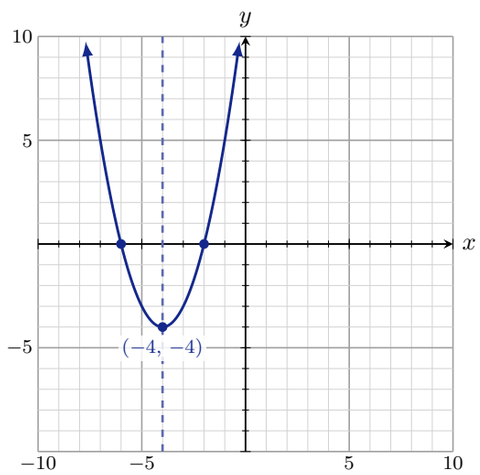
K 1 2. Use finite differences to decide whether this relation is *linear*, *quadratic*, or *neither*.

x	y	First differences	Second differences
−1	−6		
		7	
0	1		−4
		3	
1	4		−4
		−1	
2	3		−4
		−5	
3	−2		

The relation is quadratic.

Review: *Quadratic Relations and Finite Differences*

K 2 3. Sketch $y = (x+6)(x+2)$ on the grid, using its factors. Then complete each line.



Zeros: −6 and −2

Vertex: (−4, −4)

The axis of symmetry is midway between the zeros.

$x = \frac{-6+(-2)}{2}$
$= -4$

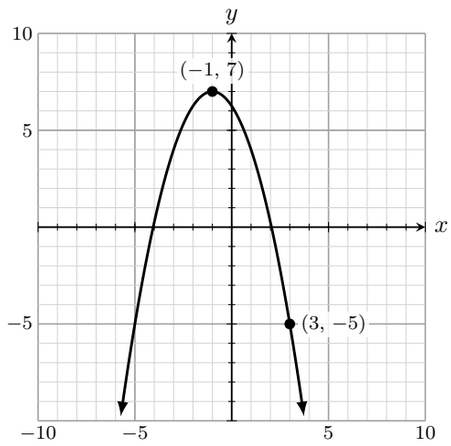
$y = (-4+6)(-4+2)$ substitute $x = -4$
$= (2)(-2)$
$= -4$

$a = 1$ is positive, so the parabola opens up.

Review: *Intercepts Form*

Marks

T 3 4. Write an equation in the form $y = a(x - h)^2 + k$ for this parabola. The vertex and one other point are marked.



The vertex is $(-1,\ 7)$, so $h = -1$ and $k = 7$.

$$\begin{aligned} y &= a(x+1)^2 + 7 \\ -5 &= a(3+1)^2 + 7 && \text{substitute } (3,\ -5) \\ -5 &= 16a + 7 && \text{subtract 7} \\ -12 &= 16a && \text{divide by 16} \\ a &= -\tfrac{3}{4} \end{aligned}$$

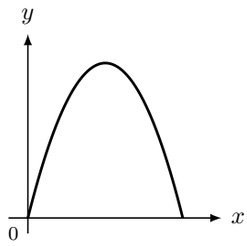
The equation is $y = -\frac{3}{4}(x+1)^2 + 7$.

Review: *Analysing Vertex Form and Equations from Graphs*

T 3 5. A fountain nozzle is at ground level. Its jet of water is modelled by

$$y = -0.5(x - 4)^2 + 8$$

where x is the horizontal distance from the nozzle and y is the height of the water, both in metres.



a) How far from the nozzle does the water land? Explain how symmetry helps. **[T 1]**

The axis of symmetry is $x = 4$. The nozzle, at $x = 0$, is 4 units left of it. The water lands 4 units right of it, at $x = 8$: 8 m from the nozzle.

b) A statue 3 m tall stands 7 m from the nozzle. Does the water pass over it? Explain. **[T 1]**

$$\begin{aligned} y &= -0.5(7-4)^2 + 8 \\ &= -0.5(9) + 8 \\ &= -4.5 + 8 \\ &= 3.5 \end{aligned}$$

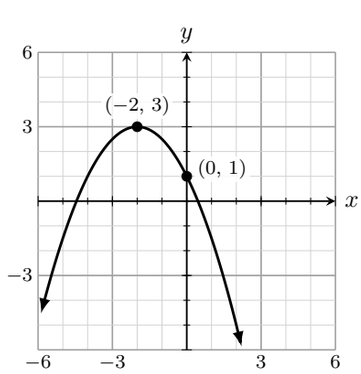
3.5 m is more than 3 m, so the water passes over it.

c) The water is 7.5 m high 3 m from the nozzle. Without substituting, at what other distance is it 7.5 m high? Explain. **[T 1]**

$x = 3$ is 1 unit left of the axis $x = 4$. $x = 5$ is 1 unit right of it, so the water is 7.5 m high again 5 m from the nozzle.

Review: *Modelling with Vertex Form and Intercepts Form*

C 2 6.



Which equation matches the graph? Explain why you chose it, and why each of the others does not match.

A $y = -\frac{1}{2}(x-2)^2 + 3$

B $y = -\frac{1}{2}(x+2)^2 + 3$

C $y = -2(x+2)^2 + 3$

The vertex is $(-2,\ 3)$, so $h = -2$ and $k = 3$. Then I find a with the point $(0,\ 1)$.

$$\begin{aligned} 1 &= a(0+2)^2 + 3 && \text{substitute } (0,\ 1) \\ 1 &= 4a + 3 && \text{subtract 3} \\ -2 &= 4a && \text{divide by 4} \\ a &= -\tfrac{1}{2} \end{aligned}$$

So B matches. A does not match: its vertex is $(2,\ 3)$, not $(-2,\ 3)$. C does not match: its a is -2, not $-\frac{1}{2}$.

Review: *Analysing Vertex Form and Equations from Graphs*

Marks

C 2 7.

x	y	First diff.
0	1	
		3
1	4	
		3
2	7	
		6
4	13	
		3
5	16	

Priya made this table and wrote: *"The first differences are not all the same, so this relation is not linear."*

Explain the flaw in Priya's reasoning.

The x-values are not evenly spaced: x jumps from 2 to 4. The 6 is the change over two steps of x, so it is 3 for each step. Every first difference is really 3, so the relation is linear.

Review: *Quadratic Relations and Finite Differences*

C 2 8. Explain why $8^0 = 1$. Use a pattern or an exponent law in your explanation.

$8^3 = 512$ divide by 8
$8^2 = 64$ divide by 8
$8^1 = 8$ divide by 8
$8^0 = 1$

Each time the exponent goes down by 1, the value is divided by 8. One more step after $8^1 = 8$ gives $8 \div 8 = 1$, so $8^0 = 1$.

Review: *Negative and Zero Exponents*

A 3 9. The main cable of a footbridge hangs between two towers 100 m apart. Each end is fixed to the top of a tower, 29 m above the deck. The lowest point of the cable is halfway between the towers, 4 m above the deck. Let x be the horizontal distance from the left tower, in metres, and y the height of the cable above the deck, in metres.

a) Write an equation in vertex form that models the cable. **[A 2]**

The vertex is $(50,\ 4)$. The left end is $(0,\ 29)$.

$y = a(x - 50)^2 + 4$
$29 = a(0 - 50)^2 + 4$ substitute $(0,\ 29)$
$29 = 2500a + 4$ subtract 4
$25 = 2500a$ divide by 2500
$a = 0.01$

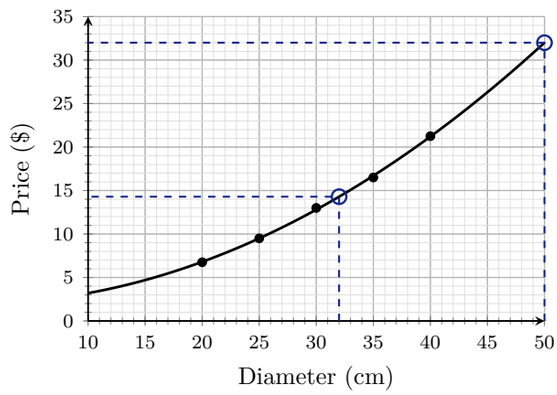
The equation is $y = 0.01(x - 50)^2 + 4$.

b) A longer bridge's cable is modelled by $y = 0.004(x - 75)^2 + 6$ (x and y as above). What do $h = 75$ and $k = 6$ tell you about it? **[A 1]**

$h = 75$: the lowest point is 75 m from the left tower. $k = 6$: the lowest point is 6 m above the deck.

Review: *Modelling with Vertex Form and Intercepts Form*

A 3 10. A pizzeria sets the price of a pizza by its diameter. Its five sizes, from 20 cm to 40 cm, are plotted below, with a curve of best fit drawn and continued to 50 cm.



a) Use the curve to estimate the price of a pizza 32 cm across and of one 50 cm across. **[A 2]**

32 cm: about \$14

50 cm: about \$32

b) Of the two diameters, 32 cm and 50 cm, which gives an *interpolation* and which an *extrapolation*? Which estimate would you trust more, and why? **[A 1]**

32 cm gives an interpolation, because 32 cm is between 20 cm and 40 cm. 50 cm gives an extrapolation, because 50 cm is beyond 40 cm. I trust the 32 cm estimate more. It is inside the data, and past 40 cm the pattern may not continue.

Review: *Relations and Models*